

7.1

Multiplying and Dividing Monomials

Focus on...

After this lesson, you will be able to...

- multiply a monomial by a monomial
- divide a monomial by a monomial



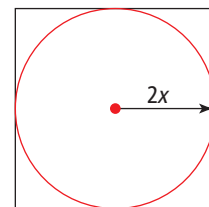
Did You Know?

The Medicine Wheel represents harmony and connection. It is an important symbol of the peaceful relationships among all living things.

Literacy Link

A *monomial* has one term. For example, 5 , $2x$, $3s^2$, $-8cd$, and $\frac{n^4}{3}$ are all monomials.

The band council would like to design a Medicine Wheel similar to the one shown above for a square area of the new school courtyard. According to the design, the edges of the outer circular pathway will just touch the edges of the square. The outer radius of the circle can be represented by $2x$. How could you determine the relationship between the area of the circle and the area of the square?



Explore Multiplying and Dividing Monomials

1. What is the side length of the square in terms of x ?
2. **a)** Write an expression for the area of the circle.
b) Write an expression for the area of the square.
3. Show how to compare the two areas using a ratio expressed in lowest terms.
4. How does the area of the square compare to the area of the circle?

Do not use an approximate value for π . Leave π in the ratio.

Reflect and Check

5. Would this relationship be the same for any circle inscribed in a square? Explain.
6. **a)** How would you multiply the monomials $4x$ and $3x$?
b) How would you divide the monomial $10x^2$ by the monomial $5x$?

Link the Ideas

Example 1: Multiply Monomials

Determine each product.

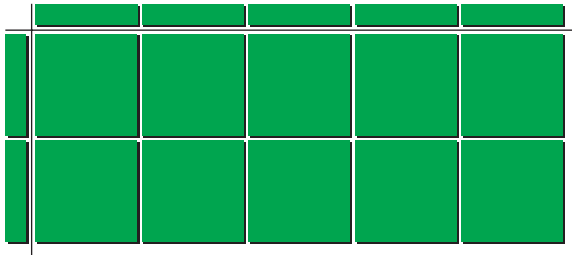
a) $(5x)(2x)$

b) $(3x)(2y)$

Solution

a) Method 1: Use a Model

You can use x -tiles and x^2 -tiles to model $(5x)(2x)$.



Each square has an area of $(x)(x) = x^2$. There are 10 positive x^2 -tiles. So, $(5x)(2x) = 10x^2$.

Method 2: Algebraically


Multiply the numerical coefficients.


Then, multiply the variables.


$$\begin{aligned} & (5x)(2x) \\ &= (5)(2)(x)(x) \\ &= 10x^2 \end{aligned}$$


How can you use the exponent laws to help you multiply the variables?

You can use algebra tiles to model algebraic expressions.

 positive x -tile

 positive y -tile

 positive xy -tile

 positive x^2 -tile

The same tiles in white represent negative quantities.

b) Method 1: Use a Model

You can use x -tiles, y -tiles, and xy -tiles to model $(3x)(2y)$.



Each grey rectangle has an area of $(x)(y) = xy$. There are 6 positive xy -tiles. So, $(3x)(2y) = 6xy$.

Method 2: Algebraically

$$\begin{aligned} & (3x)(2y) \\ &= (3)(2)(x)(y) \\ &= 6xy \end{aligned}$$

Show You Know

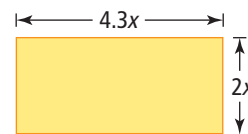
Determine each product in two different ways.

a) $(4x)(2y)$

b) $(-x)(7x)$

Example 2: Apply Monomial Multiplication

What is an expression for the area of the rectangle?



Solution

You can calculate the area, A , of a rectangle by multiplying the length by the width.

$$A = (4.3x)(2x)$$

$$A = (4.3)(2)(x)(x)$$

$$A = (8.6)(x^2)$$

$$A = 8.6x^2$$

An expression for the area of the rectangle is $8.6x^2$.

Show You Know

Calculate each product.

a) $(11a)(2b)$

b) $(-5x)(3.2)$

Example 3: Divide Monomials

Determine each quotient.

a) $\frac{-10x^2}{2x}$

b) $\frac{8xy}{4x}$

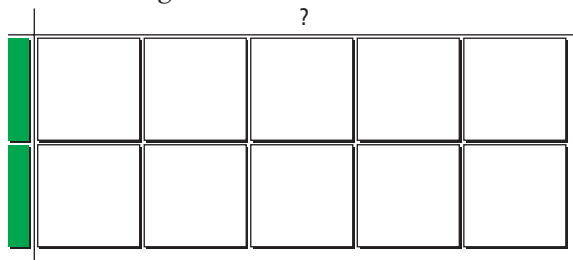
Solution

a) *Method 1: Use a Model*

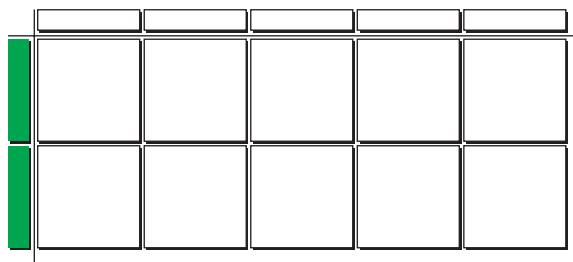
You can divide using algebra tiles.

Model $\frac{-10x^2}{2x}$ by representing the numerator with 10 negative x^2 -tiles.

Arrange the 10 tiles into a rectangle so that one of the sides is $2x$ -tiles long.



The unknown side length of the rectangle is made up of 5 negative x -tiles.



$$\frac{-10x^2}{2x} = -5x$$

Strategies

Model It

Method 2: Algebraically

You can divide the numerator and the denominator by $2x$.

$$\begin{aligned} & \frac{-10x^2}{2x} \\ &= \frac{\cancel{-10x^2}^{-5x}}{\cancel{2x}^1} \\ &= -5x \end{aligned}$$

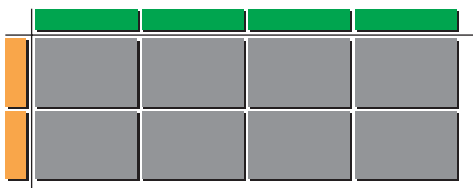
How can you use the exponent rules to help you divide the variables?

b) Method 1: Use a Model

Model $\frac{8xy}{4x}$ by representing the numerator with 8 xy -tiles. Arrange the 8 tiles into a rectangle so that one of the sides is 4 x -tiles long.



The unknown side length of the rectangle is made up of 2 y -tiles.



$$\frac{8xy}{4x} = 2y$$

Method 2: Algebraically

Divide common factors in the numerator and denominator.

$$\begin{aligned} & \frac{8xy}{4x} \\ &= \frac{\cancel{8x}^2y}{\cancel{4x}^1} \\ &= 2y \end{aligned}$$

Show You Know

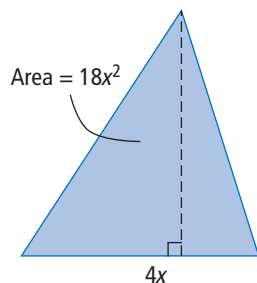
Determine each quotient.

a) $\frac{12xy}{3y}$

b) $\frac{-14x^2}{-2x}$

Strategies**Draw a Diagram****Example 4: Apply Monomial Division**

The area of a triangle is given by the expression $18x^2$. The base of the triangle is represented by $4x$. What is the height of the triangle in terms of x ?

Solution

The area of a triangle can be calculated by multiplying the base by the height, then dividing by 2.

$$\text{Area} = \text{base} \times \text{height} \div 2$$

So, if the area and base are known, then

$$\text{Height} = 2 \times \frac{\text{area}}{\text{base}}$$

$$\text{Height} = \frac{(2)(18x^2)}{4x}$$

$$\text{Height} = \frac{36x^2}{4x}$$

Divide the numerical coefficients.

Then, divide the variables.

$$\text{Height} = 9x$$

The height of the triangle is $9x$.

Show You Know

Calculate each quotient.

a) $\frac{18x^2}{3x}$

b) $14y \div (-2)$

c) $\frac{-18.6mn}{-3n}$

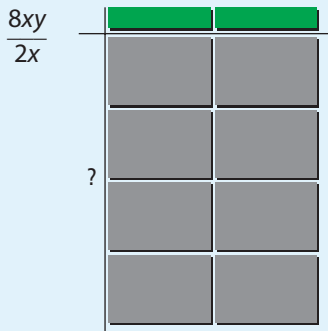
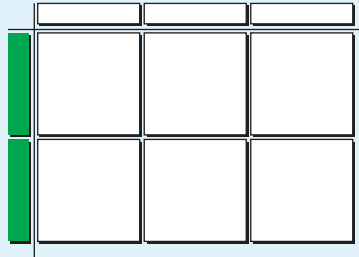
Key Ideas

- You can represent the multiplication and division of monomials using a model.

$$(2x)(-3x)$$

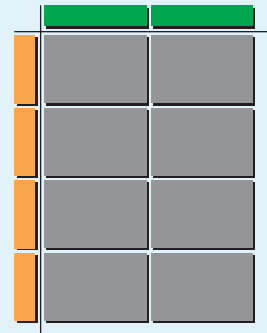
There are 6 negative x^2 -tiles.

$$(2x)(-3x) = -6x^2$$



The unknown side length of the rectangle is made up of 4 positive y -tiles.

$$\frac{8xy}{2x} = 4y$$



- To multiply monomials algebraically, you can multiply the numerical coefficients and use the exponent rules to multiply the variables.
- To divide monomials algebraically, you can divide the numerical coefficients and use the exponent rules to divide the variables.

Check Your Understanding

Communicate the Ideas

- Explain to a partner at least two ways you could find the product of $(3x)$ and $(5x)$.
- Laurie used the following method to divide $16n^2$ by $2n$.

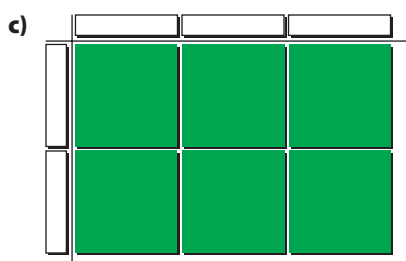
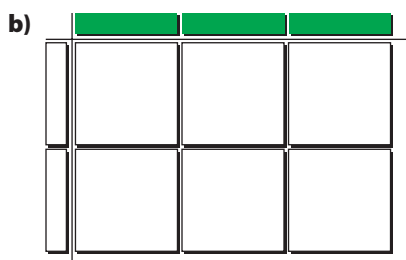
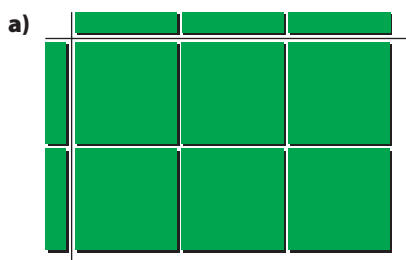
$$\begin{aligned} & \frac{16n^2}{2n} \\ & = (16 - 2)(n^2 - n) \\ & = 14n \end{aligned}$$

Does Laurie's method have any errors? If so, what are her errors and what is the correct solution?

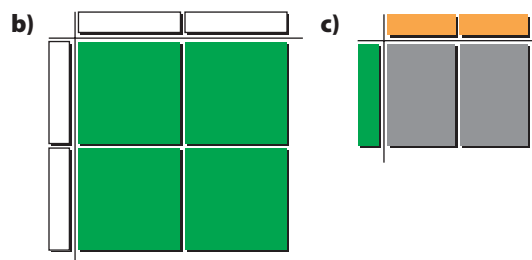
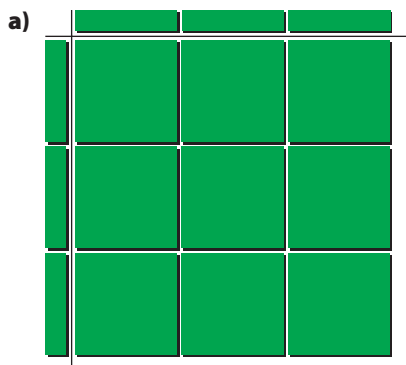
Practise

For help with #3 to #8, refer to Example 1 on page 255.

3. What multiplication statement is represented by each set of algebra tiles?



4. Determine the multiplication statement shown by each set of algebra tiles.



5. Model and complete each multiplication statement.

a) $(2x)(4x)$ b) $(-4x)(2x)$
 c) $(-4x)(-2x)$ d) $(-2x)(4x)$

6. Represent each multiplication statement with a model. Then, give the product.

a) $(3x)(5x)$ b) $(x)(-6x)$
 c) $(-3x)(2x)$ d) $(-x)(x)$

For help with #7 to #10, refer to Example 2 on page 256.

7. Find the product of each pair of monomials.

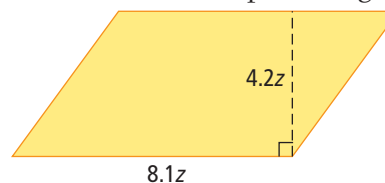
a) $(2y)(5y)$ b) $(3a)(-6b)$
 c) $(-q)(-9q)$ d) $\left(\frac{2}{3}x\right)(3x)$
 e) $(-3r)(-2t)$ f) $(1.5p)(-3p)$

8. Multiply each pair of monomials.

a) $(3n)(2n)$ b) $(-4k)(-7k)$
 c) $(-4w)(2.5w)$ d) $\left(\frac{-3}{5}x\right)(15x)$
 e) $(8m)(-0.5n)$ f) $(t)(-7t)$

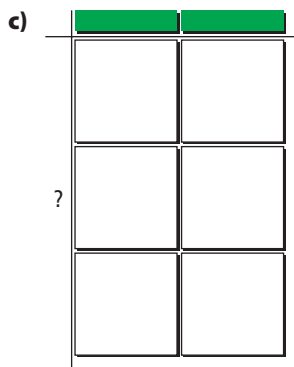
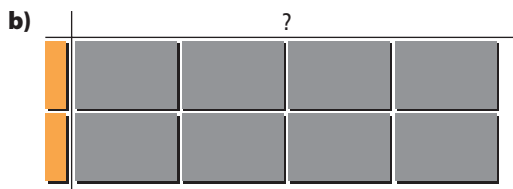
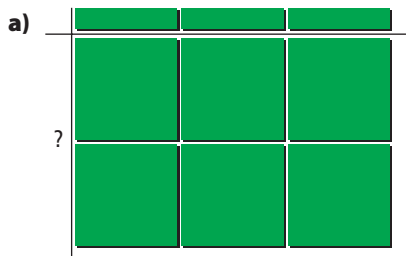
9. A rectangle has a width of $3.9x$ and a length of $5x$. What is an expression for the area of the rectangle?

10. A parallelogram has a base of $8.1z$ and a height of $4.2z$. What is an expression for the area of the parallelogram?

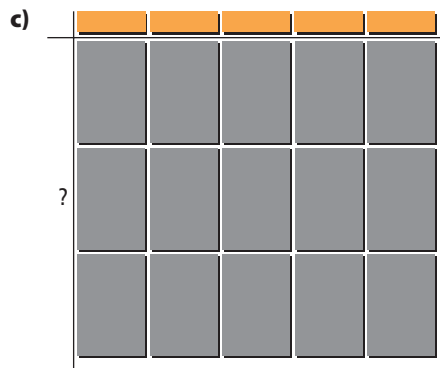
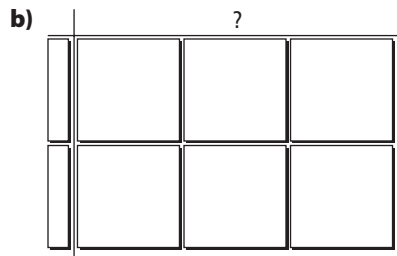
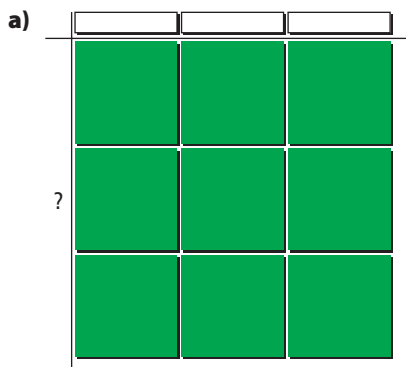


For help with #11 to #16, refer to Example 3 on pages 256-257.

11. Write the division statement represented by each set of algebra tiles.



12. Determine the division statement shown by each set of algebra tiles.



13. Model and complete each division.

a) $\frac{8x^2}{2x}$

b) $\frac{5xy}{5y}$

c) $\frac{-12x^2}{4x}$

d) $\frac{2x^2}{-x}$

14. Model and complete each division.

a) $\frac{-15x^2}{3x}$

b) $\frac{10xy}{2x}$

c) $\frac{12x^2}{-3x}$

d) $\frac{-9x^2}{-3x}$

15. Find the quotient of each pair of monomials.

a) $\frac{7x^2}{x}$

b) $\frac{25st}{5s}$

c) $\frac{125t}{5}$

d) $\frac{-8m}{-2m}$

e) $\frac{81rs}{3rs}$

f) $\frac{4.5p^2}{-3p}$

16. Divide.

a) $12.4x^2 \div x$

b) $-15r \div (-4r)$

c) $0.6t^2 \div 0.2t^2$

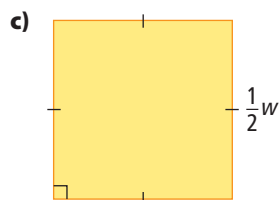
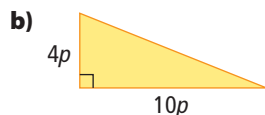
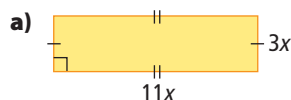
d) $-18pn \div 3n$

e) $k \div 4k$

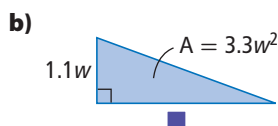
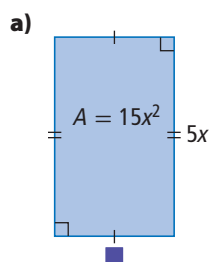
f) $\frac{2}{3}x^2 \div 2x$

Apply

17. Find an expression for the area of each figure.



18. What is the missing dimension in each figure?

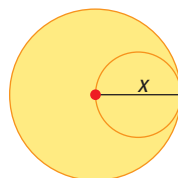


19. The area of a rectangle is $72d^2$ and its length is $20d$. What is an expression for its width?

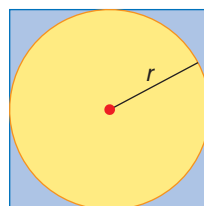
20. Claire wants to build a patio outside her café. The rectangular space outside of Claire's café is three times as long as it is wide. The area of the space is 48 m^2 . Claire would like to build a patio with dimensions 3.5 m by 12.5 m in this space. Will it fit? Explain.



21. The diagram shows that x is the radius of the large circle and the diameter of the small circle. Write the ratio of the area of the large circle to the area of the small circle. Simplify the expression.



22. A circle is inscribed in a square as shown.



In terms of the radius, r , determine each of the following ratios.

- the area of the square to the area of the circle
- the perimeter of the square to the circumference of the circle

Literacy Link

An *inscribed circle* fits exactly into another figure so that the edges of the two figures touch, but do not intersect.

23. Jonasia and Elisa are taking two tourists on a trip to photograph caribou. The visitors will be travelling by dogsled. The dogsled's length is 4 times its width. The sled has a rectangular base area of 3.2 m^2 . The equipment to be loaded on the sled measures 0.8 m wide by 3.5 m long. Will the equipment fit on the sled as it is presently packed? Explain your answer.



Extend

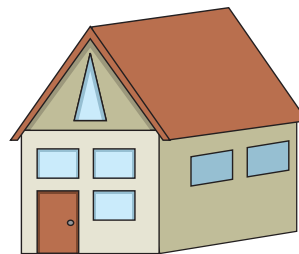
24. A rectangular prism has a volume in cubic centimetres expressed as the monomial $60xy$. The length and width of the prism, in centimetres, are $4x$ and $3y$ respectively.

- Determine the height.
- Write an expression for the surface area of the rectangular prism.

25. How is determining $\frac{9n}{3n^2}$ similar to and different from determining $\frac{9n^2}{3n}$?

26. A and B represent monomials. When A is multiplied by B, the result is A. When A is divided by B, the result is A. What is the monomial B?

27. A contractor needs to order the glass for a window. The window is in the shape of an isosceles triangle and the height of the window is 2.5 times the base width.



- Determine an expression for the area of the window in terms of the width of its base.
- If the width of the base must be 85 cm, what is the largest window area the contractor can use?

Math Link

Landscape designs for gardens may include rectangular and circular areas for flower beds, lawns, patios, and pools. As a landscape designer, you sometimes need to:

- calculate the volume of material, such as soil, gravel, water, or mulch, needed to fill these areas to a certain depth
- calculate the area that a known volume of material will cover

The following are the formulas for these calculations:

$$\text{Volume} = \text{area} \times \text{depth}$$

$$\text{Area} = \frac{\text{volume}}{\text{depth}}$$

- Draw a rectangle and circle that might be used in landscaping. Label the design element that each shape represents.
- Use variables for the dimensions of the shapes.
- Create an area formula for each shape.
- For each shape, tell what type of material you will use to fill it. Also, tell what the depth of the material will be.
- Create a volume formula for each shape.
- Along with each formula, include an explanation concerning any coefficients you use. For example, you may have to convert centimetre measurements to metres.



You may wish to create a spreadsheet that allows you to enter the values to calculate areas and volumes.

How do these coefficients relate to your landscape design?

Multiplying Polynomials by Monomials

Focus on...

After this lesson, you will be able to...

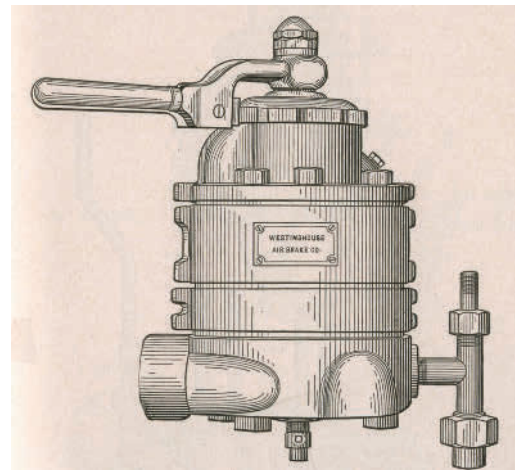
- multiply a polynomial by a monomial



In the 1880s, braking a train was a tough job. The engineer “whistled down” for brakes and reversed his engine. Two people were responsible for the brakes. One rode in the cab on the engine and the other rode in the caboos. These people would run toward each other over the swaying, rocking car tops, tightening each car’s brake control wheel as they went.

Physics Link

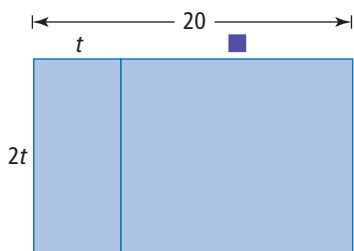
Air brakes on trains became commonplace in the 1900s, making the task of stopping a train much easier. Air brakes use compressed air in a piston to push the brake shoe onto the train wheel. The system works like a bicycle brake.



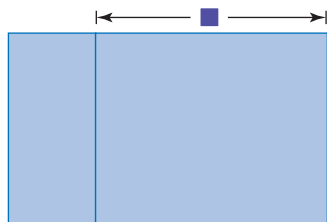
Explore Multiplying a Polynomial by a Monomial

When a train's brakes are applied, the train travels a distance before it stops. After t seconds, the distance, in metres, that the train travels is given by the polynomial $2t(20 - t)$.

1. What part of the diagram does $2t(20 - t)$ represent?



2. What polynomial represents the unknown length in the diagram? How did you determine this polynomial?



3. Find three rectangles in the diagram. What is an expression for the area of the largest rectangle? What is an expression for the area of the smallest rectangle?
4. What is the difference in area between the largest and smallest rectangles? Show two ways to find your answer.
5. Calculate the area of the medium-sized rectangle using the dimension you determined in #2.

Reflect and Check

6. Describe the steps you used in #5 to calculate the area of the medium-sized rectangle.
7. How is the area of the medium-sized rectangle related to the areas of the large rectangle and the small rectangle?
8. How far does the train travel in 10 s? Show how you arrived at your answer.

Literacy Link

A polynomial is made up of terms connected by addition or subtraction.

Examples:

$$x + 5$$

$$2d - 2.4$$

$$x$$

$$3s^2 + 5s - 6$$

$$\frac{h^2}{2} - \frac{h}{4}$$

Link the Ideas

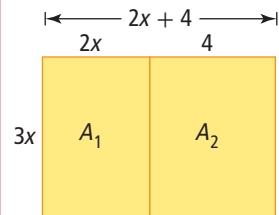
Example 1: Multiply a Polynomial by a Monomial Using an Area Model

Determine the product.

$$(3x)(2x + 4)$$

Solution

Draw a rectangle with side dimensions that represent $3x$ and $2x + 4$.



Calculate the area of each rectangle.

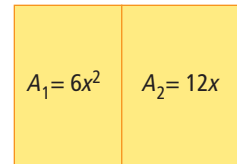
$$A_1 = (3x)(2x)$$

$$A_1 = 6x^2$$

$$A_2 = (3x)(4)$$

$$A_2 = 12x$$

The total area is $A_1 + A_2 = 6x^2 + 12x$.



Show You Know

Calculate each product.

a) $(2x)(x + 3)$

b) $(2 + c)(c)$

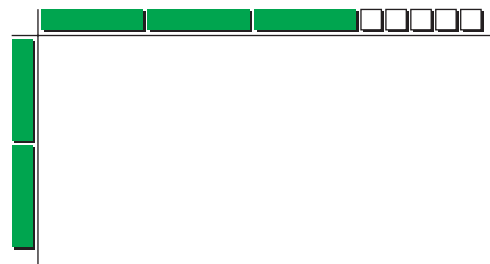
Example 2: Multiply a Polynomial by a Monomial Using Algebra Tiles

Find the product.

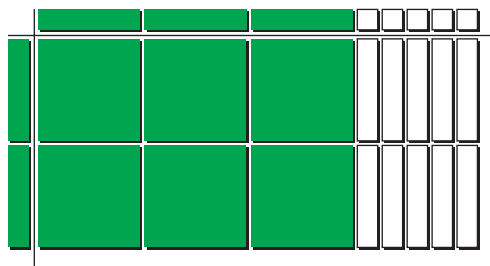
$$(2x)(3x - 5)$$

Solution

You can use x -tiles and negative 1-tiles to model $2x$ and $3x - 5$.



Use x^2 -tiles and negative x -tiles to model $(2x)(3x - 5)$.



There are 6 positive x^2 -tiles and 10 negative x -tiles.

So, $(2x)(3x - 5) = 6x^2 - 10x$.

Show You Know

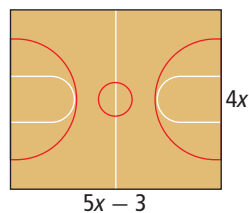
Find each product.

a) $(2 + 3x)(3x)$

b) $(4x)(2x - 1)$

Example 3: Multiply a Polynomial by a Monomial Algebraically

The dimensions of a rectangular gym floor are represented by the expressions $4x$ and $5x - 3$. What is a polynomial expression for the area of the gym floor? Write the expression in simplified form.



Solution

You can calculate the area of a rectangle, A , by multiplying the width by the length.

$$A = (4x)(5x - 3)$$

You can apply the distributive property. Multiply $4x$ by each term in the binomial $5x - 3$.

$$\begin{aligned} A &= (4x)(5x - 3) \\ A &= (4x)(5x) - (4x)(3) \\ A &= (4)(5)(x)(x) - (4)(3)(x) \\ A &= 20x^2 - 12x \end{aligned}$$

A simplified expression for the area of the gym floor is $20x^2 - 12x$.

Show You Know

Calculate each product.

a) $(-3x)(2x + 5)$

b) $(5y)(11 - x)$

Literacy Link

The *distributive property* allows you to expand algebraic expressions. Multiply the monomial by each term in the polynomial.

$$a(b + c) = ab + ac$$

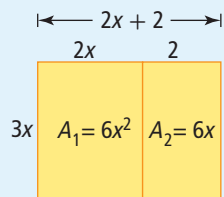
Literacy Link

A *binomial* is a polynomial with two terms, such as $6y^2 + 3$ and $2x - 5$.

Key Ideas

- You can represent the multiplication of a polynomial by a monomial using models.

- area model

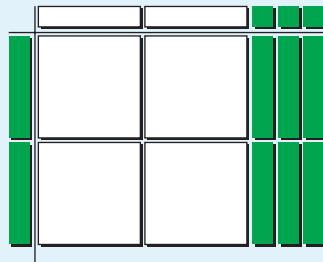


$$(3x)(2x + 2)$$

The product is represented by $A_1 + A_2$.

$$(3x)(2x + 2) = 6x^2 + 6x$$

- algebra tiles



$$(2x)(-2x + 3)$$

There are 4 negative x^2 -tiles and 6 positive x -tiles.

$$(2x)(-2x + 3) = -4x^2 + 6x$$

- To multiply a polynomial by a monomial algebraically, you can expand the expression using the distributive property. Multiply each term of the polynomial by the monomial.

$$\begin{aligned} & (-1.2x)(3x - 7) \\ &= (-1.2x)(3x) - (-1.2x)(7) \\ &= -3.6x^2 + 8.4x \end{aligned}$$

Check Your Understanding

Communicate the Ideas

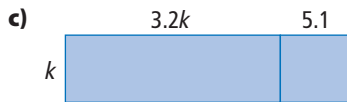
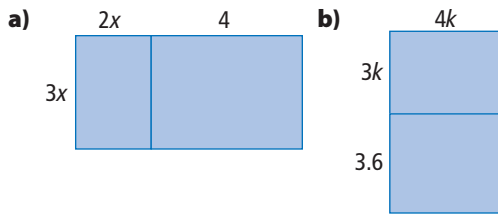
- Describe two methods you could use to multiply polynomials by monomials.
- Sara is going to simplify the expression $(3x)(2x + 4)$. Can she add the terms in the brackets and then multiply? Explain.
- Mahmoud used the following method to expand the expression $(5x)(2x + 1)$.

$$(5x)(2x + 1) = 10x^2 + 1$$
 - Show that Mahmoud's solution is incorrect.
 - How would you correct his solution?

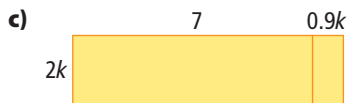
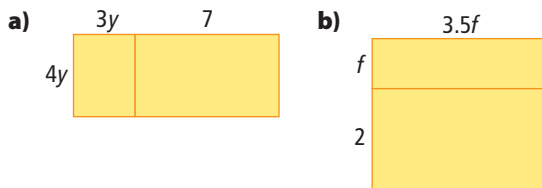
Practise

For help with #4 to #7, refer to Example 1 on page 266.

4. What multiplication statement is represented by each area model?



5. Determine the multiplication statement shown by each area model.



6. Expand each expression using an area model.

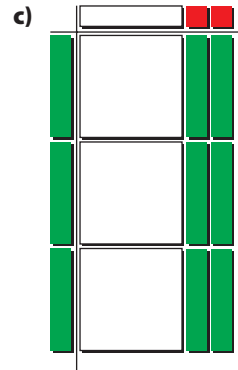
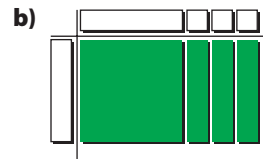
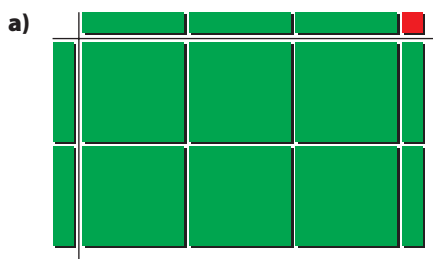
a) $(3.2r + 1)(4r)$ b) $\left(\frac{1}{2}a\right)(3a + 6)$

7. Use an area model to expand each expression.

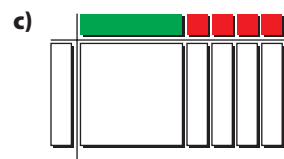
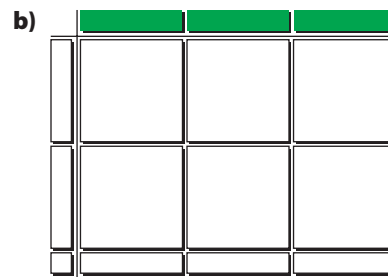
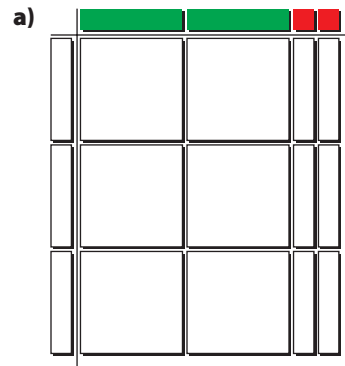
a) $(2x)(4x + 2)$ b) $(6k + 2)(4.5k)$

For help with #8 to #11, refer to Example 2 on pages 266-267.

8. What multiplication statement is represented by the algebra tiles?



9. Determine the multiplication statement shown by the algebra tiles.



10. Expand each expression, using algebra tiles.
- a) $(x - 5)(3x)$ b) $(2x)(-2x + 3)$
11. Use algebra tiles to expand each expression.
- a) $(4x + 2)(-3x)$ b) $(-4x)(3x - 1)$

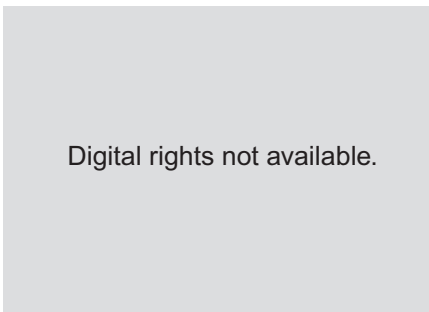
For help with #12 and #13, refer to Example 3 on page 267.

12. Expand using the distributive property.
- a) $(2x)(3x - 1)$
 b) $(3p)(2p - 0.8)$
 c) $(0.5m)(7 - 12m)$
 d) $\left(\frac{1}{2}r - 2\right)(-r)$
 e) $(2n - 7)(8.2)$
 f) $(3x)(x + 2y + 4)$

13. Multiply.
- a) $(4j)(2j - 3)$
 b) $(-1.2w)(3w - 7)$
 c) $(6x)(4 - 2.4x)$
 d) $\left(\frac{3}{7}v + 7\right)(-1)$
 e) $(3 - 9y)(y)$
 f) $(-8a - 7b - 2)(8a)$

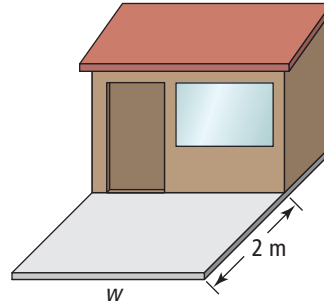
Apply

14. A rectangular Kwakiutl button blanket has a width of $3x$ and a length of $4x - 3$.



- a) What is an expanded expression for the area of the blanket?
 b) What is a simplified expression for the perimeter of the blanket?

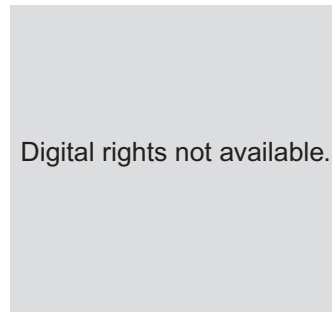
15. Lee has decided to build a shed on a square concrete slab. The shed has the same width, w , as the slab. Its length is 2 m shorter than the width of the slab.
- a) What is an expression for the area of the shed?
 b) If the width, w , of the slab is 4 m, what is the area of the shed?



16. The basketball court for the Jeux de la Francophonie is 5.5 m longer than 1.5 times the width.
- a) What is an expression for the area of the basketball court?
 b) If the length is 28 m, what is the area of the basketball court?

Did You Know?

The Jeux de la Francophonie are games for French-speaking people. They are held every four years in different locations around the world. The games include sports and artistic events. Canada is represented by three teams: Québec, New Brunswick, and a third team representing the rest of Canada.

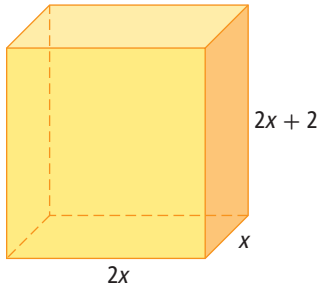


17. A rectangular field is $(4x + 2)$ m long. The width of the field is 2 m shorter than the length. What is an expression for the area of the field?

18. A rectangular skateboard park is $(3x)$ m long. Its width is 4 m less than the length.
- What is an expression for the area of the park?
 - If $x = 15$, what is the area of the park?

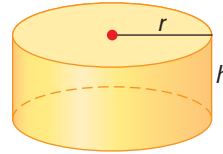
Extend

19. A rectangular packing crate has the dimensions shown, in metres.



- What is an expression for the total surface area of the crate?
- What is an expression for the volume of the crate?

20. The surface area, SA , of a cylinder is $SA = 2\pi r^2 + 2\pi rh$, where r is the radius and h is the height. The formula for the volume, V , of a cylinder is $V = \pi r^2 h$. What is the surface area of a cylinder that has a height of 5 cm and a volume of 80π cm³?



21. A rectangle measuring $(12n)$ m by 8 m has a square of side length 2 m cut out in the four corners. The cut-out shape forms an open box when the four corners are folded and taped.
- Draw the box and label its dimensions.
 - What is the surface area of this open box?
 - What is the capacity of this box?

Math Link

You are drawing up plans for a landscape design. You are going to include one of the following design elements, which will be in the shape of a rectangle:

- swimming pool
- concrete patio
- hockey rink
- beach volleyball pit



The rectangular shape is 2 m longer than twice the width. You must choose an appropriate depth for your design element.

- Create a formula for calculating the volume of material needed to fill your design element.
- Use your formula to calculate the volume of material needed for widths of 2 m, 3 m, 4 m, and 5 m. Which width would you prefer for your design element? Why?

For example, a swimming pool may have a depth of 1.5 m.

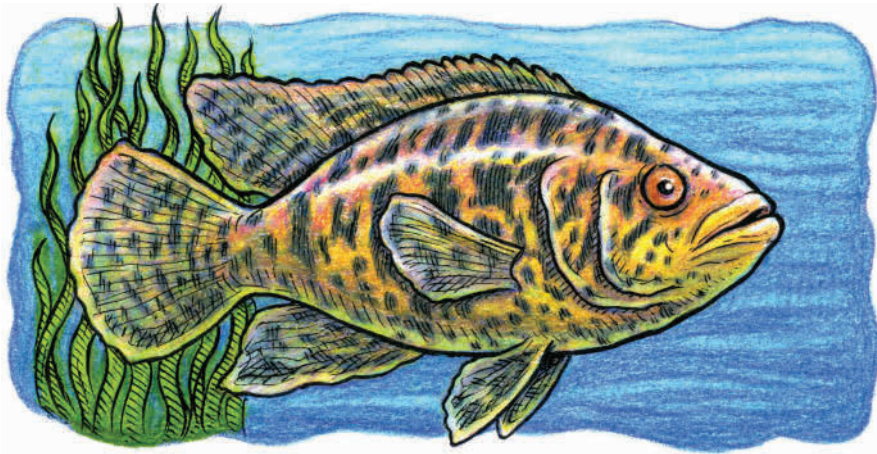
Is a pool filled right to the top with water? How might this affect your formula?

Dividing Polynomials by Monomials

Focus on...

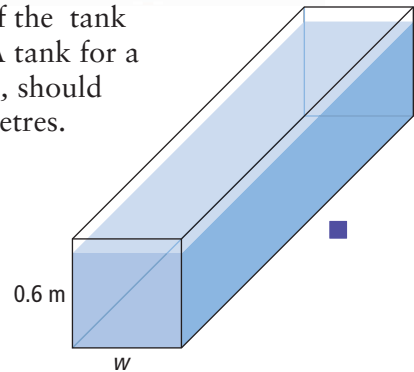
After this lesson, you will be able to...

- divide a polynomial by a monomial




When you are buying a fish tank, the size of the tank depends on the size and habits of the fish. A tank for a jaguar cichlid, or *Parachromis managuensis*, should have the minimum dimensions shown, in metres.

The volume of the rectangular tank can be represented by the polynomial expression $7.5w^2 - 3w$. How could you determine a polynomial expression that represents the length of the tank in terms of w ?



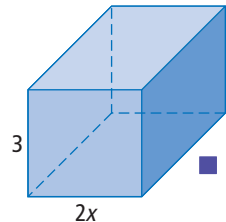
Materials

- algebra tiles 

Explore Dividing a Polynomial by a Monomial

A rectangular solid has a width of $2x$, a height of 3, and an unknown length. The area of the base of the solid is represented by the polynomial $2x^2 + 4x$.

1. Show that the volume of the solid shown can be represented by the polynomial $6x^2 + 12x$.
2. Use algebra tiles to represent the area of the rectangular base.
3. Count the number of x -tiles and 1-tiles required for the missing dimension of the rectangle. What expression represents the missing dimension?



What happens if you multiply your expression by the width of the rectangular solid?

Reflect and Check

4. Show that your expression for the missing dimension in #3 is correct.
5. Describe the steps you would take to find an expression for the ratio of the volume to the side measuring $2x$.

Link the Ideas

Example 1: Divide a Polynomial by a Monomial Using a Model

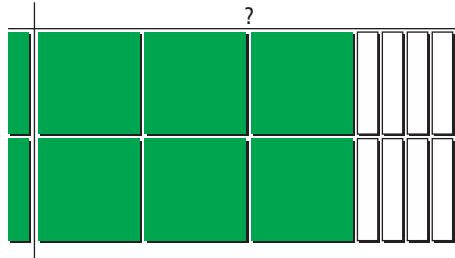
Determine the quotient.

$$\frac{6x^2 - 8x}{2x}$$

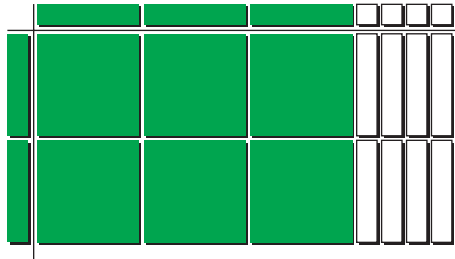
Solution

You can use algebra tiles. Use 6 positive x^2 -tiles and 8 negative x -tiles to represent the polynomial $6x^2 - 8x$.

The vertical side of the rectangle represents the monomial divisor, $2x$.



Count the number of positive x -tiles and negative 1-tiles required to complete the horizontal side of the rectangle.



There are 3 positive x -tiles and 4 negative 1-tiles, or $3x - 4$. This expression represents the result of dividing the polynomial, $6x^2 - 8x$, by the monomial, $2x$.

$$\frac{6x^2 - 8x}{2x} = 3x - 4$$

Check:

Multiply the quotient, $3x - 4$, by the divisor, $2x$.

$$\begin{aligned} & (2x)(3x - 4) \\ &= (2x)(3x) - (2x)(4) \\ &= 6x^2 - 8x \end{aligned}$$

How do you know that the answer is correct?

You can also determine the quotient algebraically.

$$\begin{aligned} & \frac{6x^2 - 8x}{2x} \\ &= \frac{6x^2}{2x} - \frac{8x}{2x} \\ &= \frac{\overset{3x}{\cancel{6x^2}}}{\underset{1}{\cancel{2x}}} - \frac{\overset{4}{\cancel{8x}}}{\underset{1}{\cancel{2x}}} \\ &= 3x - 4 \end{aligned}$$

Strategies

Model It

Show You Know

Determine each quotient.

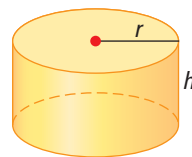
a) $\frac{3x^2 + 6x}{3x}$

b) $\frac{8x^2 - 2x}{2x}$

The formula for the surface area of a cylinder is $2\pi r^2 + 2\pi rh$.

Example 2: Dividing a Polynomial by a Monomial Algebraically

- a) What is the ratio of the surface area to the radius of the cylinder? Write the ratio in simplified form.
- b) If the height, h , of the cylinder is the same as the radius, r , what is the ratio of the surface area to the radius? Write the ratio in simplified form.



Solution

a)
$$\frac{\text{surface area}}{\text{radius}} = \frac{2\pi r^2 + 2\pi rh}{r}$$

The expression can be broken down into two parts.

$$\begin{aligned} & \frac{\text{surface area}}{\text{radius}} \\ &= \frac{2\pi r^2}{r} + \frac{2\pi rh}{r} \\ &= \frac{2\pi \overset{r}{\cancel{r^2}}}{\underset{1}{\cancel{r}}} + \frac{2\pi \overset{1}{\cancel{r}}h}{\underset{1}{\cancel{r}}} \\ &= 2\pi r + 2\pi h \end{aligned}$$

- b) Substitute $h = r$ into the ratio from part a).

$$\begin{aligned} & \frac{\text{surface area}}{\text{radius}} \\ &= 2\pi r + 2\pi h \\ &= 2\pi r + 2\pi(r) \\ &= 2\pi r + 2\pi r \\ &= 4\pi r \end{aligned}$$

Show You Know

Determine each quotient.

a)
$$\frac{15x^2 - 12x}{3x}$$

b)
$$\frac{-2t^2 + 4t}{2t}$$

Key Ideas

- You can divide a polynomial by a monomial using a model.

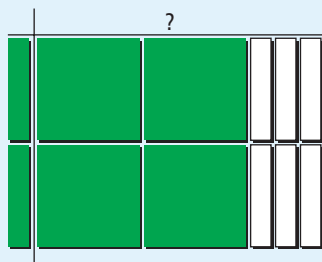
$$\frac{4x^2 - 6x}{2x}$$

The unknown side length of the rectangle is made up of $2x - 3$ tiles.

$$\frac{4x^2 - 6x}{2x} = 2x - 3$$

- When you divide a polynomial by a monomial algebraically, you can divide the numerical coefficients and apply the exponent laws to the variables.

$$\begin{aligned} & \frac{4x^2 - 8x}{2x} \\ &= \frac{4x^2}{2x} - \frac{8x}{2x} \\ &= \frac{\overset{2x}{\cancel{4x^2}}}{\underset{1}{\cancel{2x}}} - \frac{\overset{4}{\cancel{8x}}}{\underset{1}{\cancel{2x}}} \\ &= 2x - 4 \end{aligned}$$



Check Your Understanding

Communicate the Ideas

1. Explain how you would perform the following division: $\frac{3x^2 + 6x}{2x}$.

2. Anita used the following method to simplify an expression:

a) Show that Anita's solution is incorrect.

b) How would you correct her solution?

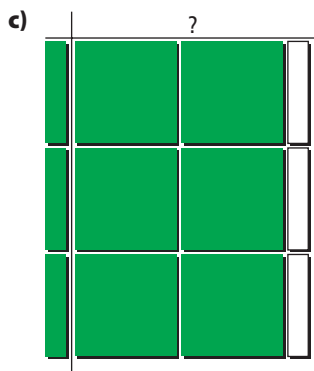
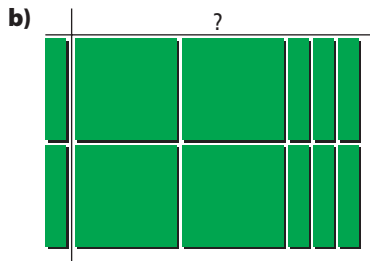
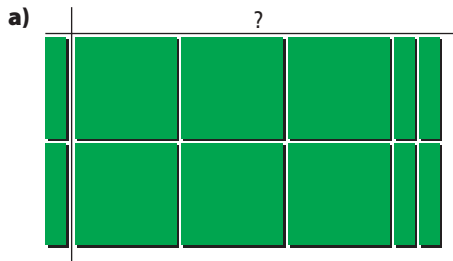
$$\begin{aligned} & \frac{9k^2 - 3k}{3} \\ &= \frac{9k^2}{3} - \frac{3k}{3} \\ &= 3k - 1 \end{aligned}$$

3. Use a model to show a polynomial division statement with a quotient of $3x + 2$.

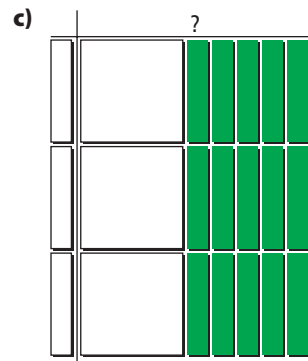
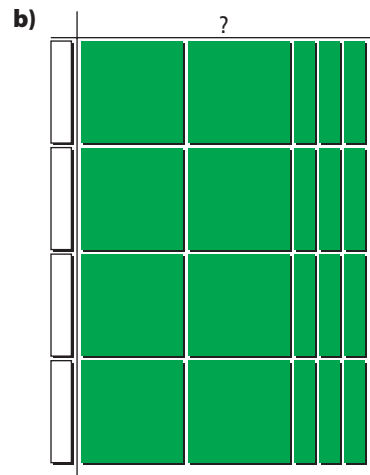
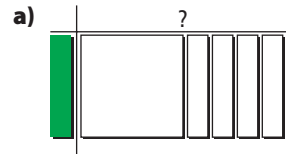
Practise

For help with #4 to #7, refer to Example 1 on page 273.

4. What division statement is represented by the algebra tiles? Determine the quotient.



5. Determine the division statement represented by the algebra tiles and give the quotient.



6. Divide each expression, using a model.

a) $\frac{5x^2 - 10x}{5x}$

b) $\frac{4x^2 + 12x}{2x}$

7. Use a model to divide each expression.

a) $\frac{-8x^2 - 4x}{4x}$

b) $\frac{-3x^2 + 5x}{-x}$

For help with #8 and #9, refer to Example 2 on page 274.

8. Divide.

a) $\frac{2y^2 + 4.2y}{2y}$

b) $\frac{12m^2 - 6.2m + 24}{2}$

c) $\frac{-18y^2 - 6y}{-6y}$

d) $\frac{3cv - 2.7c}{3c}$

9. Determine each quotient.

a) $\frac{2.7c^2 + 3.6c}{3c}$

b) $\frac{2x^2 + 8xy}{x}$

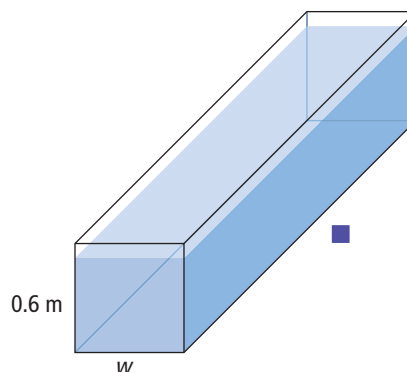
c) $\frac{-s^2 - 1.5st}{5s}$

d) $\frac{-14w^2 - 7w + 0.5}{0.5}$

Apply

10. A dump truck holds 10 m^3 of soil. You are filling a rectangular space in a yard with the dimensions of $(2x + 3)$ by $5x$ by 2 , in metres. What polynomial expression represents the number of truck loads of soil you will need?

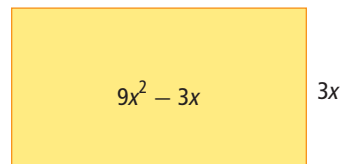
11. A rectangular fish tank has the dimensions shown, in metres. The volume of the tank can be represented by $7.5w^2 - 3w$.



- What polynomial expression represents the area of the base of the tank?
- What polynomial expression represents the length of the tank?
- What is the length of the tank if the width is 0.6 m ? What is the volume of the tank?

12. For their Valentine's Day dance, the grade 9 students want to decorate the end wall of the gym with red poster paper. The area of the wall is given by the polynomial $45x^2 + 20x$. One sheet of poster paper covers an area given by the monomial $5x$. What polynomial expression represents the number of sheets of paper the students will need to cover the wall?

13. A rectangle has an area of $9x^2 - 3x$ square units. The width of the rectangle is $3x$ units. What is the length?



14. The formula used to predict the distance an object falls is $d = 4.9t^2 + vt$. In the formula, d is the distance, in metres, t is the time, in seconds, and v is the starting velocity of the object, in metres per second.

a) The average speed of a falling object is calculated as $s = \frac{d}{t}$, where s is the average speed, in metres per second. Use this information to develop a formula for the average speed of a falling object in terms of t and v .

b) What is the average speed of an object that falls for 5 s, if it starts from a resting position?

Extend

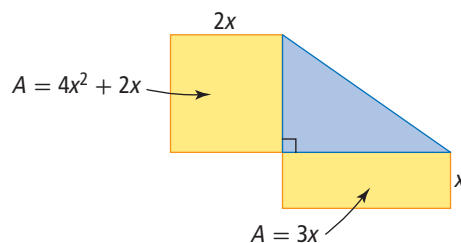
15. Divide.

a)
$$\frac{3.6gf + 0.93g}{0.3g}$$

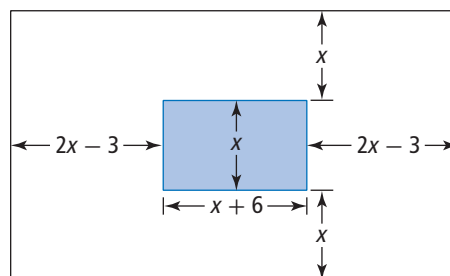
b)
$$\frac{\frac{2}{3}b^2 - \frac{1}{3}ab + \frac{1}{3}b}{\frac{1}{3}b}$$

c)
$$\frac{-4.8x^2 + 3.6x - 0.4}{0.2}$$

16. Two rectangles have common sides with a right triangle, as shown. The areas and widths of the rectangles are shown. What is a simplified expression for the area of the triangle?



17. What is the ratio of the area of the shaded rectangle to the area of the large rectangle?



18. If a rectangle has length $2xy$ and area $12x^2y + 6xy^2$, what is its perimeter?

Math Link

You are designing a park that includes a large parking lot that will be covered with gravel.

- Design two different-shaped parking lots using any single shape or combination of regular shapes. Include the dimensions on a drawing of each parking lot design. Note that you will need to be able to calculate the area of your parking lots. Each area should be a different shape. Make them no less than 200 m^2 and no greater than 650 m^2 .
- A truck with dimensions similar to those shown in the picture will deliver the gravel. Write an expression for the approximate area that a single load of gravel will cover to a depth of 5 cm.
- There are three sizes of trucks that can deliver the gravel. The widths are 1.5 m, 2 m, and 3 m. Approximately how many truckloads would it take for each truck size to deliver the required amount of gravel for each of your parking lots? You will cover each parking lot to a depth of 5 cm. Show your work.
- Which truck size do you think would be the most efficient to use for each of your parking lots? Explain your reasoning.



Chapter 7 Review

Key Words

For #1 to #4, match the polynomial in Column A with an equivalent polynomial in Column B. Polynomials in Column B may be used more than once or not at all.

Column A

1. $\frac{8xy}{2x}$

2. $\frac{12x^2 - 6x}{3x}$

3. $(-2x + 1)(-2x)$

4. $\frac{12xy - 6x}{3}$

Column B

A $4xy - 2x$

B $4x^2 - 2x$

C $4y$

D $2x^2 - 2x$

E $4xy$

F $4x - 2$

7.1 Multiplying and Dividing Monomials, pages 254–263

5. Use a model to complete each monomial multiplication statement.

a) $(3x)(5x)$

b) $(4x)(-5y)$

6. Find each product.

a) $(-3.2x)(-2.7y)$

b) $\left(\frac{3}{7}a\right)(-14a)$

7. Use a model to complete each monomial division statement.

a) $\frac{6x^2}{2x}$

b) $15a^2 \div (-3a)$

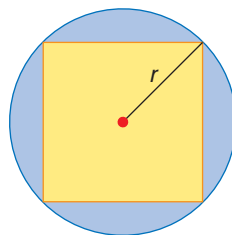
8. Determine each quotient.

a) $\frac{-4.8r^2}{-1.2r}$

b) $2xy \div 2x$

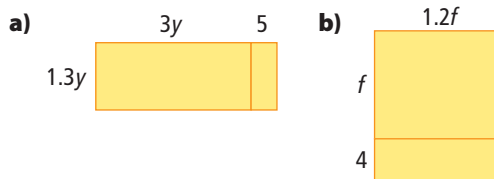
9. A rectangle is four times as long as it is wide. If the area of the rectangle is 1600 cm^2 , what are its dimensions?

10. A square is inscribed in a circle with radius r as shown. What is the ratio of the area of the square to the area of the circle?

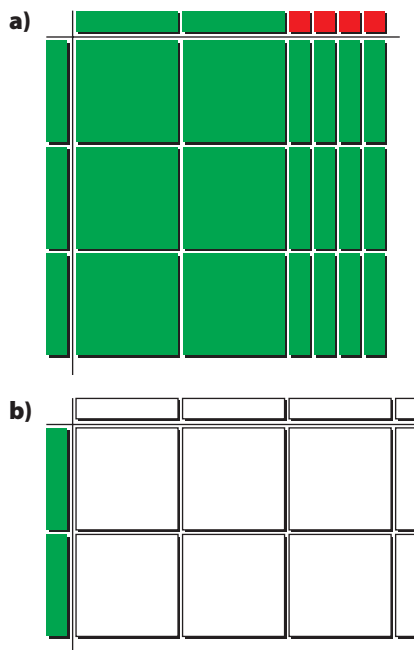


7.2 Multiplying Polynomials by Monomials, pages 264–271

11. What polynomial multiplication statement is represented by each area model?



12. What polynomial multiplication statement is represented by the algebra tiles?

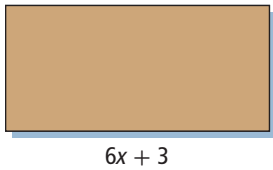


13. Expand.

a) $(20x)(2.3x - 1.4)$

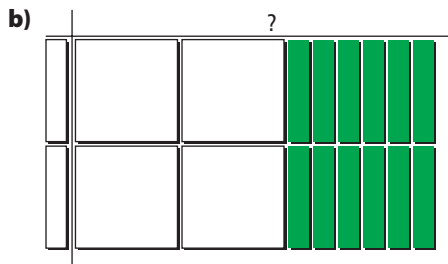
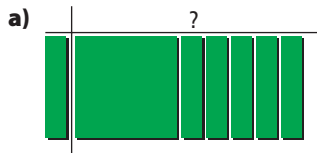
b) $\left(\frac{2}{3}p\right)\left(p - \frac{3}{4}\right)$

14. The length of a piece of rectangular cardboard in centimetres is $6x + 3$. The width is 1 cm less than $\frac{1}{3}$ of the length. What is an expression for the area of the cardboard?



7.3 Dividing Polynomials by Monomials, pages 272-277

15. Determine the division statement represented by the algebra tiles. Give the quotient.

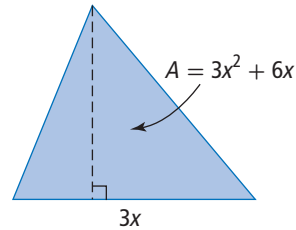


16. Divide.

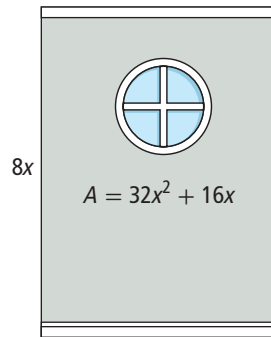
a) $\frac{12n^2 - 2n}{2n}$

b) $\frac{15x - 3x^2}{1.5x}$

17. A triangle has an area represented by $3x^2 + 6x$. If the base of the triangle is $3x$, what is the height?



18. A rectangular wall has a circular window. The area of the wall can be represented by $32x^2 + 16x$. The length of the wall is $8x$. The diameter of the circular window has a measurement that is half the width of the wall. What is the radius of the window written as an expression in terms of x ?



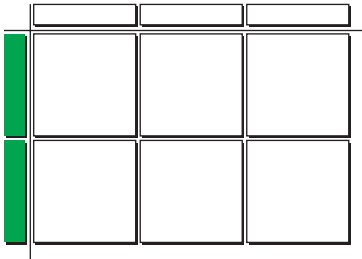
19. Naullaq is cutting ice blocks from the lake for her mother's drinking water tank. The cylindrical tank has a volume of $4x^2\pi$. Once each block has melted, it will have a volume of $3x^2$. How many blocks does she need to cut so that her mother's tank will be filled when the ice melts? Give your answer to the nearest whole block. Explain your answer.

Use the π key on your calculator.

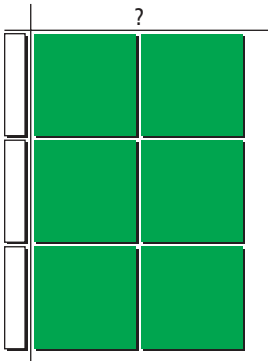
Chapter 7 Practice Test

For #1 to #6, select the best answer.

1. Which monomial multiplication statement is represented by the algebra tiles?



- A $(3x)(-2x) = -6x^2$
 B $(2x)(-3x) = -6x^2$
 C $(2x)(3x) = 6x^2$
 D $(-2x)(-3x) = 6x^2$
2. What is the product of $3y$ and $2.7y$?
- A $0.9y$ B $8.1y$
 C $0.9y^2$ D $8.1y^2$
3. Which monomial division statement is represented by the algebra tiles?



- A $\frac{-6x^2}{-3x} = -2x$
 B $\frac{-6x^2}{-3x} = 2x$
 C $\frac{6x^2}{-3x} = -2x$
 D $\frac{6x^2}{-3x} = 2x$

4. Which is equivalent to $-27q^2 \div 9q$?
- A $3q^2$ B $3q$
 C $-3q$ D $-3q^2$
5. Which is equivalent to $(\frac{2}{3}x)(-3x - 6)$?
- A $-2x^2 - 4x$ B $-2x - 4$
 C $2x - 4$ D $2x^2 - 4x$
6. Calculate $\frac{15y^2 - 10y}{-5y}$.
- A $-3y - 2$ B $-3y + 2$
 C $3y - 2$ D $3y + 2$

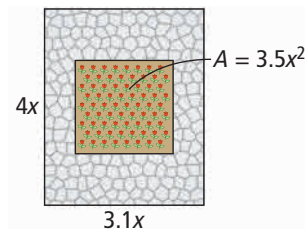
Complete the statements in #7 and #8.

7. The expression $\frac{-24x^2 + 8xz}{4x}$ is equivalent to \blacksquare .
8. A polynomial multiplication expression that is equivalent to $24d^2 - 12d$ is \blacksquare .

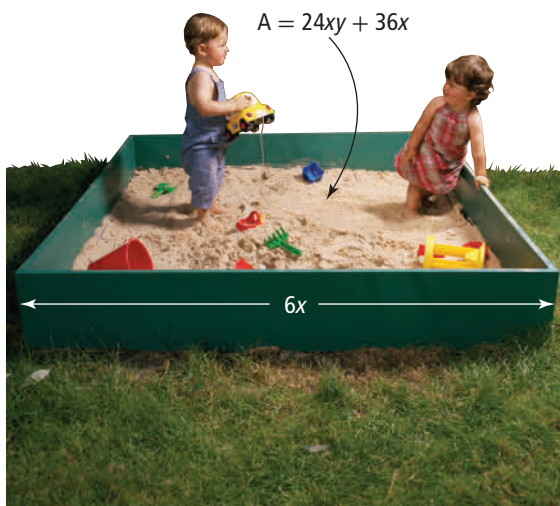
Short Answer

For #9 to #11, show all of the steps in your solutions.

9. Calculate $(2.4x)(4y)$.
10. What is the product of $12h$ and $\frac{-3}{4}h + 2$?
11. Simplify $\frac{2x^2 + 3x}{-3x}$.
12. Paula is building a rectangular patio. It will have a square flower bed in the middle. The rest will have paving stones. The patio will have a length of $4x$ and a width of $3.1x$. The area of the flower bed will be $3.5x^2$. What area of the patio will need paving stones?



13. A sports field is 15 m longer than twice its width. What is an expression for the area of the field in terms of its width, w ? Expand the expression.
14. The area of a rectangular sandbox can be expressed as $24xy + 36x$. The width of the sandbox is $6x$. What is the perimeter of the sandbox?



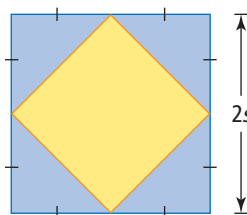
Extended Response

15. a) What error did Karim make when completing the division statement shown?

$$\begin{array}{r} -18d^2 - 6d \\ \underline{3d} \\ -18d^2 + 6d \\ \underline{3d} \\ -6d + 2 \end{array}$$

- b) Show the correct method.

16. A square with a side length of $2s$ has a smaller square inscribed. The vertices of the smaller square are at the midpoints of the sides of the larger square. What is the ratio of the area of the larger square to the area of the smaller square? Express your answer in its simplest form.



Math Link: Wrap It Up!

You have been hired to create a landscape design for a park. The park is rectangular and covers an area of $500\,000\text{ m}^2$. The park includes the following features:

- a play area covered with bark mulch
- a sand area for playing beach volleyball
- a wading pool

The features in your design include the following shapes:

- a circular area
- a rectangular area
- a parallelogram-shaped area with the base three times the height

The features of your park have varying depths.

Include the following in your design:

- a scale drawing showing the layout of each of the required features
- a list showing the area of each feature and the volume of each material (mulch, sand, and water) required to complete the park
- a polynomial expression for the area and volume of each feature, using a variable for one of the dimensions



Challenges

Design a Card Game

You are a game designer. You are going to create a skill-testing card game that involves the multiplication and division of polynomial expressions.



Materials

- at least 30 index cards per pair of students, or heavy paper for cutting out cards

Work with a partner. You will create at least 15 pairs of cards, using the following guidelines:

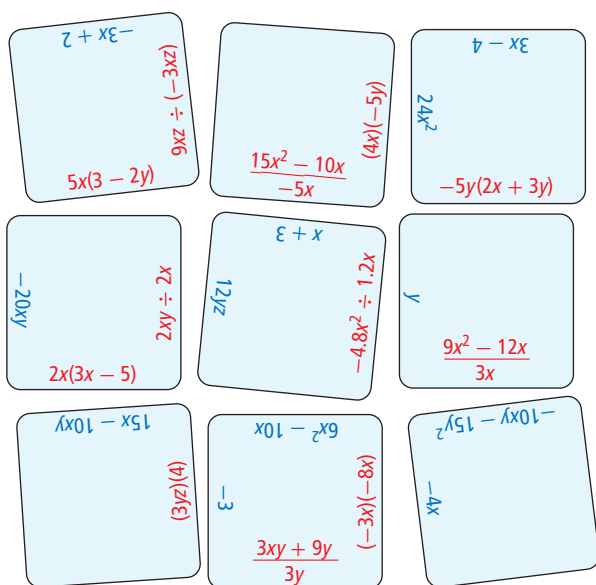
- For each pair, on one card write a polynomial expression that involves multiplication or division. On the other card write an equivalent expression.
- Include the types of polynomial expressions found in each section of Chapter 7.
- Use a variety of constants, monomials, and polynomials.

1. Create a list of the pairs of expressions for your game.
2. Create the cards.
3. Write the rules for your game. Your game should be suitable for two or more players.
4. Exchange your cards and rules with another set of partners. Play the game.

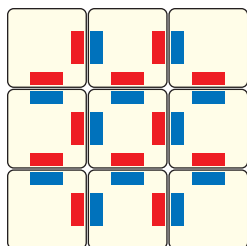
One card might be $(5x)(4x)$, and the equivalent card might be $20x^2$.

Polynomial Puzzle

1. Try the nine-piece puzzle below.



- Cut out the pieces of a copy of the puzzle.
- Solve the puzzle. Polynomial expressions involving multiplication or division are **red**. Matching equivalent expressions are **blue**. Match each multiplication or division expression with its equivalent expression. The diagram shows how a solved nine-piece puzzle would look.



2. Design your own 16-piece puzzle.

- Draw 16 equal-sized squares on a piece of paper.
- Write your own matching expressions. Place them the same way as in the small diagram above so that each expression and its equivalent are across from each other.
 - Include the types of polynomial expressions found in Chapter 7.
 - Include a variety of constants, monomials, and polynomials.
 - Cover a variety of difficulty levels.
- Cut out your sixteen puzzle pieces. Mix them up.
- Exchange your puzzle with a classmate's. Solve your classmate's puzzle.

Materials

- sample polynomial puzzle per student
- scissors

Chapters 5–7 Review

Chapter 5 Introduction to Polynomials

1. The following diagram of algebra tiles models a backyard.



- a) What is an expression for the perimeter of the backyard?
 b) What is an expression for the area of the backyard?

2. Use materials or diagrams to show the collection of like terms in each expression.

- a) $8c + 3 - 5c + 1$
 b) $-1 + x - 1 - x + 1$
 c) $g^2 - g + 5 + 2g - 4g^2$

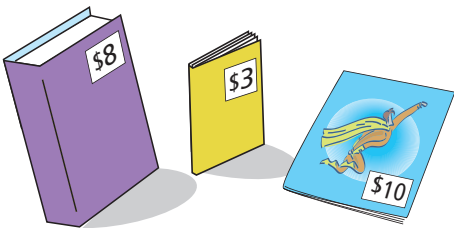
3. Write the following expressions in simplest form.

- a) $(2m - 3) + (5m + 1)$
 b) $(w^2 - 4w + 7) + (3w^2 + 5w - 3)$
 c) $(9y^2 - 6.8) + (4.3 - 9y - 2y^2)$

4. Write a simpler expression.

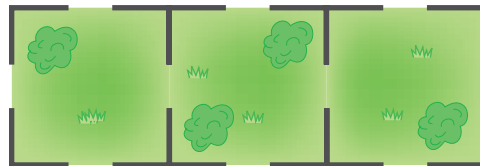
- a) $(-7z + 3) - (-4z + 5)$
 b) $(3d - 2d - 7) - (d^2 - 5d + 6cd - 2)$
 c) $(2x^2 + 3xy) - (-xy + 4x^2)$

5. The Better Buys antique shop sells comic books for \$10, hardcover books for \$8, and paperback novels for \$3.



- a) Write an algebraic expression for the antique shop's total income from the sale of comics, hardcover books, and paperbacks. Tell what each variable represents.
 b) Use your expression to show the total income after the sale of 15 comics, 7 hardcover books, and 5 paperbacks.
 c) One day, the store sold \$100 worth of comics, hardcovers, and paperbacks. What number of each item did the store sell? Show that more than one answer is possible.

6. A park is divided equally into three square sections. Each section will have a side measurement of $2n + 4$. The park will have fencing built as shown. Each opening has length n and does not need any fencing. What is the total length of fencing needed to complete the job?



Chapter 6 Linear Relations

7. a) Describe the relationship between the figure number and the number of tiles.

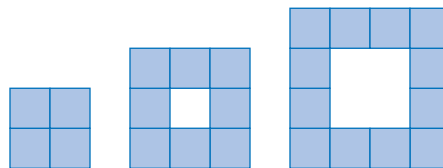


Figure 1

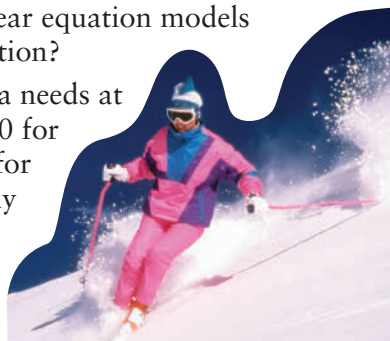
Figure 2

Figure 3

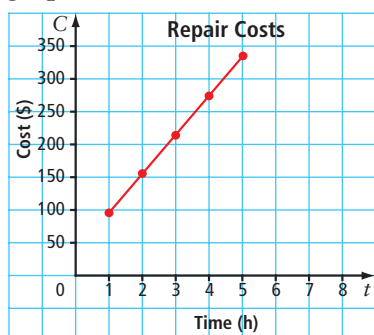
- b) Develop a linear equation to model the pattern.
 c) If the pattern were continued, how many squares would be in Figure 8?

8. Monika is saving money for a ski trip. She starts with \$112 in her bank account. She decides to deposit \$25 every week until she has enough money to pay for the trip.

- Create a table of values for the first five deposits.
- What linear equation models this situation?
- If Monika needs at least \$450 for her trip, for how many months does she need to deposit money into her bank account?



9. A car mechanic charges a \$35 base fee for labour plus an hourly rate of \$60. The graph shows this linear relation.



- Approximately how much would the mechanic charge after working on a vehicle for 8 h?
- Approximately how many hours would a mechanic work to charge \$225 in labour costs?
- Another mechanic charges at the same rate, but in half-hour increments. What would be the cost of a repair that took 9.5 h to complete?



10. A computer salesperson earns a monthly salary plus a 10% commission on each sale. The sales, commission, and earnings are shown in the table.

Sales (\$)	Commission (\$)	Earnings (\$)
0	0	2000
5 000	500	2500
10 000	1000	3000
15 000	1500	3500
20 000	2000	4000
25 000	2500	4500

- Draw a graph showing the linear relation between sales and earnings.
- The salesperson earns \$3750 in one month. What are the approximate total sales for that month?
- The salesperson earns \$5500 the next month. What are the approximate total sales for that month?
- Approximately how much would the salesperson have to sell in order to earn \$4250 in one month?

11. Draw the graph that represents this table of values.

Time (h)	0.25	0.50	0.75	1.00	1.25	1.50	1.75
Cost (\$)	0.5	1.0	1.5	2.0	2.5	3.0	3.5

- Describe a situation to represent the data on the graph.
- Write an equation to model the data.
- What is the cost for 3.25 h?

Chapter 7 Multiplying and Dividing Polynomials

12. Find the product of each pair of monomials.

- a) $(3x)(4x)$
- b) $(2.5y)(-4y)$
- c) $(s)(-0.5s)$
- d) $\left(\frac{t}{5}\right)(10t)$

13. Divide.

- a) $8.4x^2 \div x$
- b) $(-12h^2) \div 2h$
- c) $(-0.6n^2) \div (-0.2n)$
- d) $\frac{4.8p^2}{-1.2p}$

14. Use an area model to expand each expression.

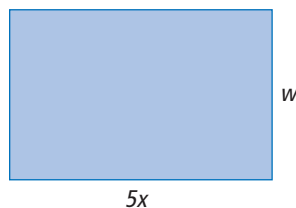
- a) $(3x)(2x + 1)$
- b) $(5w + 3)(1.5w)$

15. If a foosball table is 3 cm longer than twice its width, what is an expression for the area of the foosball table? Express your answer in expanded form.

16. Determine each quotient.

- a) $\frac{12g^2 + 8g}{4g}$
- b) $\frac{-6x^2 + 3xy}{3x}$
- c) $(9.3ef^2 - 62e) \div (-3.1e)$
- d) $(24n^2 + 8n) \div (0.5n)$

17. A rectangle has an area represented by the expression $10x^2 - 5x$. If the length of the rectangle is $5x$, what is an expression for the width, w , of the rectangle in terms of x ?



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Choosing a Television to Suit Your Room

You want to find a television that

- best suits your needs, and
- considers your room size and the location for the television.

Materials

- measuring tape
- grid paper

Does a standard or high-definition television (HDTV) make the most sense for your room? How large of a screen should you get?

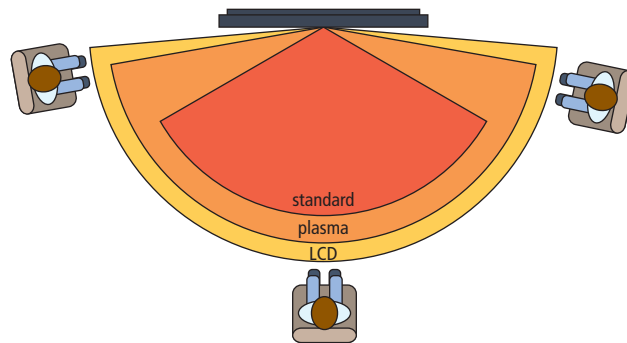
- The following table gives you the best viewing distance for the screen size for two types of TVs.

Screen Size (cm)	Viewing Distance (cm)	
	Standard TV	HDTV
68.8	205.7	172.7
81.3	243.8	203.2
94.0	281.9	233.7



- Given this information, what size of television would be best for your classroom? Make a sketch of your classroom, including where you plan to place the TV and the best place for a student to view it from.
- If the television is 320 cm away from your seat, how large of a standard TV would be best?
- How will your answer for part b) change if you have a HDTV?

- The diagram shows the viewing angles for various types of televisions. Calculate the viewing area of the TV type and size of your choice.



Viewing angle:
 standard: 120°
 plasma: 160°
 LCD: 170°

- What type and size of TV would be best for a room in your home? Justify your response.